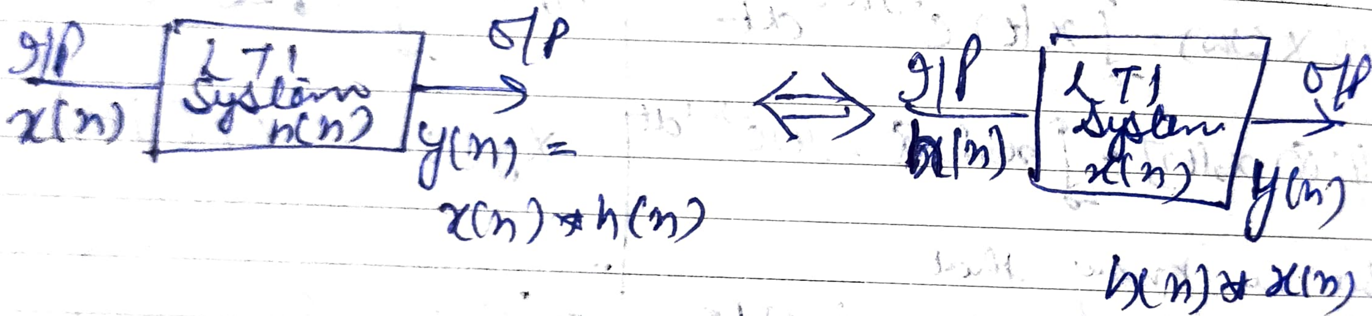
Properties of Linear Convolution

① Commutative
② Associative Property
③ Distributive Property
④ Static and dynamic
⑤ Invertability
⑥ Causality
⑦ Stability
⑧ unit step Response

① Commutative

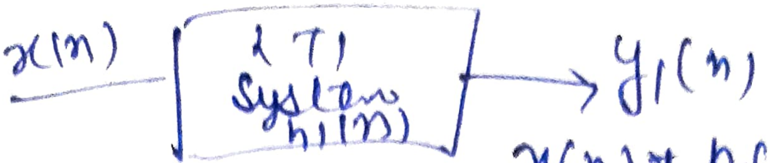
$$x(n) * h(n) = h(n) * x(n)$$



② Associative

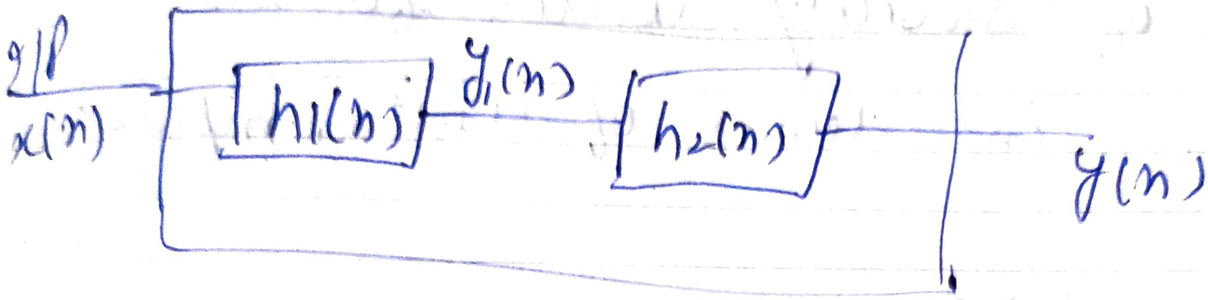
$$[x(n) * h_1(n)] * h_2(n)$$

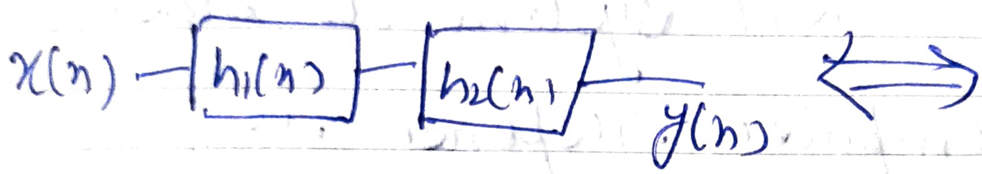
$$x(n) * [h_1(n) * h_2(n)]$$

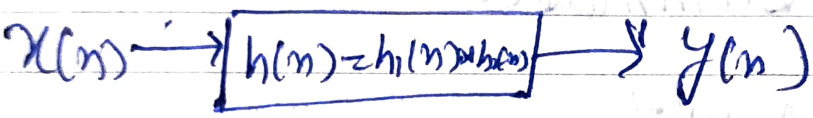


$x(n) * h_1(n)$

$y(n) = y_1(n) * h_2(n)$

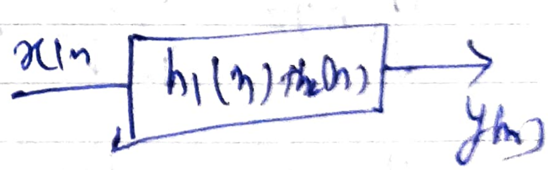






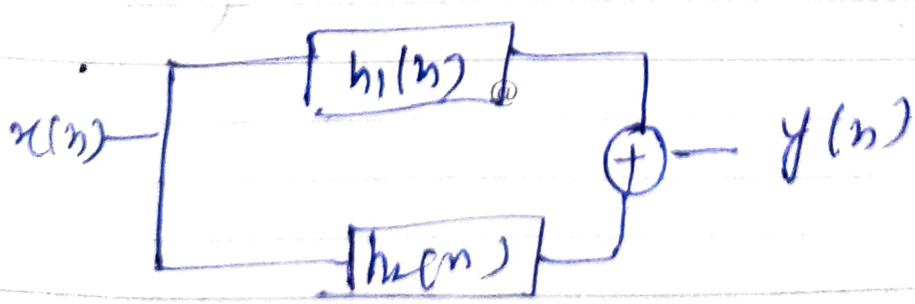
$x(n) * [h_1(n) * h_2(n)]$

Distributive Property →



$x(n) * [h_1(n) + h_2(n)] =$

$[x(n) * h_1(n)] + [x(n) * h_2(n)]$



APRIL 2015

Wk	M	T	W	T	F	S	S
14			1	2	3	4	5
15	6	7	8	9	10	11	12
16	13	14	15	16	17	18	19
17	20	21	22	23	24	25	26
18	27	28	29	30			

Properties of LTI System →

(1) Stability

Proof of Stability Criteria for LTI system in terms of unit Impulse

Statement ⇒ LTI System is stable if its impulse response is absolutely summable.

Proof

$h(n)$ Impulse response
$x(n)$ I/P

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k) \quad \text{——} (1)$$

It is also written as

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k) \quad \text{——} (2)$$

1^{st} we will define bounded sequence.

Let us denote this finite no. by M_x

$$|x(n)| \leq M_x \quad \text{—(3)}$$

$$|x(n)| \leq M_x < \infty \quad \text{—(4)}$$

Now taking absolute value of both side of eq^n (2)

$$|y(n)| = \left| \sum_{k=-\infty}^{\infty} h(k)\, x(n-k) \right| \quad \text{—(5)}$$

absolute value of sum of items

If we take Σ sign outside the term becomes

$$\sum_{k=-\infty}^{\infty} |h(k)|\, |x(n-k)| \quad \text{—(6)}$$

Summation of absolute value of terms

Thus $$\left| \sum_{k=-\infty}^{\infty} h(k)\, x(n-k) \right| \leq \sum_{k=-\infty}^{\infty} |h(k)|\, |x(n-k)| \quad \text{—(7)}$$

@

but in eq (5) the LHS is $y(n)$ so eq (7) can be written as

$$|y(n)| \leq \sum_{k=-\infty}^{\infty} |h(k)|\,|x(n-k)| \quad (8)$$

Here $x(n-k)$ is delayed input signal.

If i/p is bounded then its delayed version is also bounded.

$$|x(n)| \leq M_x$$

$$|x(n-k)| \leq M_x$$

Put this value in eqn (8)

$$|y(n)| \leq \sum_{k=-\infty}^{\infty} |h(k)| \cdot M_x \quad (9)$$

$$\therefore |y(n)| \leq M_x \sum_{k=-\infty}^{\infty} |h(k)| \quad (10)$$

We know that M_x is a finite no, we want o/p $y(n)$ to be bounded. That means $|y(n)|$ should be finite. So from eqn (10) to obtain finite o/p we have

$$\sum_{k=-\infty}^{\infty} |h(k)| < \infty \quad (11)$$

Here, $h(k) = h(n)$ is the impulse response of LTI system. So (11) gives the condition of stability in terms of impulse response of the system.

Now stability factor is denoted by S

$$\therefore S = \sum_{k=-\infty}^{\infty} |h(k)| < \infty$$

Thus LTI system is stable if its impulse response is absolutely summable.

Causality of LTI System →

discrete time system is causal if its output is depends only on present and past value I/P & not on future I/p. We will prove that LTI system is causal if its impulse response $h(n) = 0$ for $n < 0$

A LTI system is causal if its impulse response $h(n) = 0$ for -ve value of n

Proof →

According to the equation of linear convolution we have

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k) \quad \text{--- (1)}$$

written as

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k) \quad \text{--- (2)}$$

limit express into two parts

$$y(n) = \sum_{k=0}^{\infty} h(k)\, x(n-k) + \sum_{k=-\infty}^{-1} h(k)\, x(n-k) \quad \text{(3)}$$

now expand the summation

$$y(n) = h(0)\,x(n) + h(1)\,x(n-1) + h(2)\,x(n-2) \text{ --- }$$
$$+\, h(-1)\,x(n+1) + h(-2)\,x(n+2) \text{ --- --- ---}$$

Let us consider present condition as at $n=0$

h(0) so $x(n)$ present
$x(n-1)$ past
$x(n+1)$ future

MARCH 2015

Wk	M	T	W	T	F	S	S
09	30	31					1
10	2	3	4	5	6	7	8
11	9	10	11	12	13	14	15
12	16	17	18	19	20	21	22
13	23	24	25	26	27	28	29

@

Thus for Causality

$h(-1) = h(-2) = h(-3) = 0$

That means

$$h(n) = 0 \text{ for } n<0$$

Thus for causal LTI system the eq'n of linear convolution is

$$y(n) = \sum_{k=0}^{\infty} x(k)\, h(n-k)$$

OR

$$y(n) = \sum_{k=0}^{\infty} h(k)\, x(n-k)$$

Numerics

Unit 12